Penetration with Step Frequency Ground Penetrating Radar

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Step frequency radar equation

The signal to noise ratio of an ultra wide band ground penetrating radar is given by:

\[ SNR = \frac{P \tau G_t G_r \lambda_1 \lambda_2 \sigma}{(4 \pi)^3 R^4 k T 10^{\alpha R}} \]  

(1)

where \( P \) is the transmitted power, \( \tau \) is the integration time, \( G_t \) and \( G_r \) are the transmit and receive antenna gains, \( \lambda_1 \) and \( \lambda_2 \) are the longest and shortest wave lengths respectively, \( \sigma \) is the target radar cross section, \( R \) is the target range, \( k \) is Boltzmann’s constant, \( T \) is the effective receiver noise temperature and \( B \) is the receiver bandwidth. In the case of GPR, the signal attenuation in the ground has exponential character, hence the exponential loss factor. For the development of this equation, please see “Development of the ultra wide band step frequency radar equation“ at the end of this paper.

Note that, in practice, also the target radar cross section will vary with frequency. This is not accounted for in the above equation. Also, the signal attenuation in the ground is usually (but not always) strongly frequency dependent.

Since the power and gain are constant, the range for a given target cross section depends mainly on integration time and the soil attenuation. The same is true for SNR:

\[ SNR \sim \frac{\tau}{R^4 10^{\alpha R}} \]  

(2)

\[ SNR_{\text{dB}} \sim 10 \log \tau - 40 \log R - 10 \alpha R \]  

(3)

The image below shows the signal variance with respect to depth (represented by time) of a certain area. The flat area to the right is where the signal variance is less than the noise, this the total signal + noise becomes flat, since the noise floor is independent of depth. The low area to the left is where the signal return is from above the ground surface. The signal level is constant in this area, so the variance is low. Note that this curve does not represent the total dynamic range, since the variance of the topmost targets is much lower than the power from those. The maximum signal level, is roughly 20 dB higher than the variance for this data set, so the largest SNR is roughly 80 dB in this case.

Illustration 1: Var(S+N) for a typical GPR data set
The interesting part of this curve is the slope from approximately 7 ns to about 50-60 ns. This gradient illustrates the \(-40 \log R\) relationship of the signal level which is typical whenever the range dependency is more significant than the soil attenuation. The point where this gradient crosses the horizontal noise floor to the right, we call the “Knee Point”. This is a fairly good indicator of the relative penetration (when comparing different integration times). In this case, the “Knee Point” is at approximately 60 ns.

**The relationship between penetration and survey speed**

Whenever the GPR is used with normal soil (case 1 above), the penetration may be increased by increasing the signal energy per area.

3d-Radar Ground Penetrating Radar has fixed output power set by local legal emission levels. Therefore, the only way to increase the signal energy is to increase the integration time. The integration time is the time spent sending the entire frequency range for a single trace:

\[
\tau = t_{\text{dwell}} N_{\text{freq}} = t_{\text{dwell}} \frac{BW}{\Delta f}
\]

where \(t_{\text{dwell}}\) is the dwell time (the time spent on each frequency), \(N_{\text{freq}}\) is the number of distinct frequencies in a trace, \(BW\) is the total bandwidth and \(\Delta f\) is the frequency step. In the client, the frequency step is not set directly. Instead there is a setting for “Time Window”. The time window is defined as half of the ambiguity range for a given frequency step, so the relationship becomes:

\[
\tau = t_{\text{dwell}} t_{\text{win}} \frac{2BW}{\Delta f}
\]

where \(t_{\text{win}}\) is the Time Window.

Increased integration time can be achieved by either increasing the dwell time or increasing the Time Window or a combination of both. Increased integration time will result in improved penetration, but do note: minor adjustments will go unnoticed! In order to double the penetration, roughly 16 times increase in integration time is needed!

One of 3d-Radar strong points is the market leading work rate. An area may be surveyed in less time than with any other system, partly due to the array nature of the antennae and partly due to the high rate of advance possible. The limiting factor is the integration time. Due to the dual receiver design on the GeoScope mkIV, two channels are acquired simultaneously. But even then, a full scan of all the channels, will take some time. The total time to scan all channels to create a cross line slice of data is:

\[
t_{\text{scan}} = \frac{n_{\text{ch}} \tau}{2} + t_{\text{overhead}}
\]

where \(n_{\text{ch}}\) is the number of antenna channels, \(\tau\) is the integration time and \(t_{\text{overhead}}\) is the time for channel switching, calibration etc. The factor 1/2 is due to the dual receiver design. In order to be able to complete the scan before the next distance trigger, the rate of advance must be lower than the maximum speed:

\[
v_{\text{max}} = \frac{\Delta x}{t_{\text{scan}}}
\]

where \(\Delta x\) is the horizontal sample spacing in the direction of advance. This maximum speed is reported in the client based on the user settings of Trigger Interval, Time Window and Dwell Time.

**Example**

In order to demonstrate the speed / penetration relationship, we made several surveys over the same road section using a DX1821 antenna. In order to demonstrate the full range, we used a fairly long trigger distance of 30 cm. With this sampling interval, the maximum speed (using the lowest possible integration time) would be 391 kph, a speed which the antenna could probably not achieve
even in free fall. The following figures show the signal power variance plots for various settings.  
Once again, please note that the max S/N is about 20 dB higher than the Var(S+N) shown.

Illustration 2: Var(S+N). Dwell Time = 0.7 µs, Frequency step = 20 MHz (25 ns Time Window). Max speed 391 km/h

Illustration 3: Var(S+N). Dwell Time = 2 µs, Frequency step = 10 MHz (50 ns Time Window). Max speed 96 km/h

Illustration 4: Var(S+N). Dwell Time = 10 µs, Frequency step = 10 MHz (50 ns Time Window). Max speed 21 km/h

Illustration 5: Var(S+N). Dwell Time = 2 µs, Frequency step = 2 MHz (250 ns Time Window). Max speed 19 km/h.  
(The differing channels are 4 extra long offset channels)

Illustration 6: Var(S+N). Dwell Time = 10 µs, Frequency step = 2 MHz (250 ns Time Window). Max speed 4 km/h.

As seen in the illustrations above, the maximum Var(S+N)/N varies with integration time, but what about penetration? Although not being an absolute measure, the “Knee Point” of the curves, where the Var(S+N) equals N is a good indicator of the penetration. This is illustrated below.
The importance of soil conditions

Of all the factors that affect the GPR penetration, soil conditions is by far the most important one. There are soil conditions where the signal energy (Power multiplied with integration time) is the only limit, and the soil behaves almost like free space. On the other hand, there are soil conditions where the soil is completely impenetrable by GPR and the penetration depth will be zero regardless of signal energy. In the latter case, GPR does not work. Nothing to do about it.

Let us study three major cases:

Case 1: Normal attenuation soil
When radio waves penetrate normal soil (meaning: none of the cases above), the soil will attenuate the signal to some extent. This attenuation is partly due to absorption (conversion to heat) and partly due to the signal being scattered from numerous mini-reflectors in its path. The total attenuation is strongly dependent on the soil, and thus, so is the penetration. In most cases, the signal attenuation is also dependent on the signal frequency in that lower frequencies receive less attenuation than higher frequencies. Increased signal energy will result in increased penetration. GPR works.

Case 2: Highly conductive soil
When the radio waves reach a conductive surface, it will be reflected. As the resistivity of the material approaches zero, the amount of signal reflected approaches 100 %. The radio waves will not penetrate the top surface and zero penetration into the material is the result. Metal plates are, of course, a typical material where the penetration is zero, but also materials like marine clay will produce such results due to the combination of salt and water. In a radargram, such material will show up with a strong reflection from the top surface and no signal below. GPR does not work.

Case 3: Highly absorptive soil
When radio waves reach a material which have high absorption, the energy is quickly depleted. Resistive materials and also ferromagnetic materials behave this way. These materials behave much like the induction stove, in that all of the signal energy is converted to heat in the first couple of millimeters. Zero penetration beyond that point is the result. Increase of signal energy does not help. In a radargram, such materials are often hard to spot as the surface reflection is usually very low and there is no signal below. GPR does not work.

In the normal case (Case 1), the signal attenuation is strongly soil dependent. Usually (but not always) the attenuation will increase when the signal frequency increases. The figure below illustrates the frequency dependency for three typical soil conditions:
Illustration 8: Signal attenuation vs frequency

**Frequency Range versus Penetration**

Illustration 8 shows the attenuation versus frequency. In most cases the attenuation will increase with increasing frequency, but not always. We will consider the two cases shortly, but before we do that, we need to visit one of the radar fundamentals:

The range resolution (in time) of any radar is the inverse of the Signal Bandwidth:

$$\rho = \frac{1}{BW} \quad (8)$$

Where \(\rho\) is the -3dB resolution (in time) and BW is the -3dB signal bandwidth. This relation is a fundamental relation for all radars. The resolution does not depend on signal frequency (as some believe), but on signal bandwidth.

Now, let us consider the two attenuation cases:

**Attenuation increases with increasing frequency (most usual case):**
Low frequency receives less attenuation than the higher frequencies. Thus, the bandwidth of the signal received from the deep is smaller than the bandwidth of the transmitted signal. There are two consequences of this:

1) The resolution will decrease with increasing depth (due to reduced bandwidth)

2) The Signal Bandwidth will decrease with depth, but the Noise Bandwidth will not

Deeper layers or objects will show up with reduced resolution and the ability to separate objects or layers that are close in depth will degrade with depth. The fact that the signal bandwidth decreases while the noise bandwidth does not, will result in reduced Signal to Noise Ratio (SNR) and therefore reduced penetration. This can easily be mitigated in the post processing by reducing the signal bandwidth in the processing steps (before transforming into time domain). Since the reflected signal is already reduced in bandwidth, doing so will only reduce the noise bandwidth. The SNR will improve as long as the processed signal bandwidth is higher or equal to the reflected signal bandwidth. Beyond that point, reducing the processing bandwidth will result in poorer resolution without improving the penetration significantly.

**Attenuation does not increase with increasing frequency (e.g. dry sand):**
Low frequencies receives the same attenuation as the higher frequencies. Thus the bandwidth of the signal is constant and equal to transmitted signal. The resolution is more or less constant for all depths. Reducing the bandwidth in processing will not result in improved penetration and will have a negative impact on resolution. This situation generally coincides with low attenuation in general, thus the penetration is already very good. Reducing the processing bandwidth will have detrimental effect on the data.
Development of the ultra wide band step frequency radar equation

The signal to noise ratio of any radar is given by:

\[
\text{SNR} = \frac{PG_tG_r\lambda^2\sigma}{(4\pi)^3R^4kTB}L
\]  

(9)

where \(P\) is the transmitted power, \(G_t\) and \(G_r\) are the transmit and receive antenna gains, \(\lambda\) is the wavelength, \(\sigma\) is the target radar cross section, \(R\) is the target range, \(k\) is Boltzmann's constant, \(T\) is the apparent receiver noise temperature and \(B\) is the receiver bandwidth. The loss factor, \(L\), accounts for all signal loss. In the case of GPR, the signal attenuation in the ground has exponential character:

\[
L = 10^\alpha R
\]

(10)

When the receiver filter is matched to the transmitted signal and coherent reception is performed, the relation becomes:

\[
\text{SNR} = \frac{P\tau G_tG_r\lambda^2\sigma}{(4\pi)^3R^4kT}10^\alpha R
\]

(11)

where \(\tau\) is the integration time. Note that the bandwidth is no longer part of the equation.

The 3d-Radar ground penetrating radar creates an ultra wide band signal by stepping through the total bandwidth, one frequency at a time. Thus, there is no single wave length in the system. In order to use the radar equation for such a signal, we need to replace the term \(\lambda^2\).

Assuming constant antenna gains, we can remedy this by splitting the total signal into a series of signals with constant frequency from \(f_1\) to \(f_2\) (not including the latter) using a frequency step of \(\Delta f\). Each frequency transmitted a time period \(t_{\text{dwell}}\). Setting \(f_1=n_1t_{\text{dwell}}\) and \(f_2=n_2t_{\text{dwell}}\), we arrive at the relation:

\[
\sum t_{\text{dwell}}\lambda^2 \approx \sum \frac{t_{\text{dwell}}\Delta f^2}{\Delta f} \frac{1}{x^2} \approx \frac{t_{\text{dwell}}\Delta f^2}{\Delta f} \frac{1}{n_1-n_2} = \frac{t_{\text{dwell}}\Delta f^2}{\Delta f} \frac{1}{f_1-f_2}
\]

(12)

Substituting the total integration time, \(\tau\) for \((n_2-n_1)t_{\text{dwell}}\), and \((f_2-f_1)\) for \((n_2-n_1)\Delta f\) gives us a result, that may be substituted in the original radar equation:

\[
\sum t_{\text{dwell}}\lambda^2 \approx \tau \frac{c^2}{f_2-f_1} \left( \frac{1}{f_1} - \frac{1}{f_2} \right) = \tau \frac{c^2}{f_2f_1} = \tau \lambda_2 \lambda_1
\]

(13)

where \(\lambda_1\) and \(\lambda_2\) are the longest and shortest wave lengths respectively. In the case of the 3d-Radar system the wave lengths varies between 3m and 0.1m, corresponding to 100MHz and 3 GHz transmit frequency.

The ultra wide band radar equation becomes:

\[
\text{SNR} = \frac{P\tau G_tG_r\lambda_1\lambda_2\sigma}{(4\pi)^3R^4kT}10^\alpha R
\]

(14)